Math 1160 Section 5.1 Key
Spring 200

1. Problem 1

   (a) We have only one subinterval in this instance. The length of this subinterval is given by $\Delta t = \frac{9-6}{1} = 3$. So our estimates are

   $$\begin{aligned} \text{Left estimate} &= 3 \cdot 100 = 300\ m^3 \quad \text{(underestimate)} \\ \text{Right estimate} &= 3 \cdot 280 = 840\ m^3 \quad \text{(overestimate)} \end{aligned}$$

   To get the best estimate, average these two values.

   $$\text{Best estimate} = \frac{\text{Left} + \text{Right}}{2} = \frac{300 + 840}{2} = 570\ m^3/hr.$$

   (b) We know that the flow rate in increasing throughout the whole time so we can say that the difference between the under- and over-estimates (the error) is

   $$\text{Error} \leq \Delta t\,(280 - 100) = 180\Delta t.$$

   We need to choose the value of Δt so that the error $180\Delta t \leq 6$. Solving for Δt we get $\Delta t \leq 1/30$. What this means is that the flow gauge should be read every $1/30$ of an hour, or every 2 minutes.

2. Problem 2

   (a) Lower estimate $= (45)\,(2) + (16)\,(2) + (0)\,(2) = 122$ feet.
   Lower estimate $= (88)\,(2) + (45)\,(2) + (16)\,(2) = 298$ feet.

3. Problem 5

   (a) Car A

   (b) Car A

   (c) Car B since the area under the curve that represents his velocity is much greater than the area under the other curve.

4. Problem 7

   (a) We need to compute the left- and right-hand sums. Since the car is decreasing throughout, we know that the left-hand sum will give us an overestimate while the right hand sum will give us an underestimate. Since the length of each interval is 2 seconds, we have $\Delta t = 2$ and so

   $$\begin{aligned} \text{Left estimate} &= 2\,(100 + 80 + 50 + 25 + 10) = 530 \text{ ft.} \\ \text{Right estimate} &= 2\,(80 + 50 + 25 + 10 + 0) = 330 \text{ ft.} \end{aligned}$$

   The best estimate of the distance traveled will be the average of these two estimates, or

   $$\text{Best estimate} = \frac{530 + 330}{2} = \frac{860}{2} = 430 \text{ ft.}$$

   (b) We can't tell wheter or not we capped the skunk or not.

5. Problem 9
   Using $\Delta t = 0.2$, the upper estimate is

   $$0.2\left(\frac{1}{1+0} + \frac{1}{1+0.2} + \frac{1}{1+0.4} + \frac{1}{1+0.6} + \frac{1}{1+0.8}\right) \approx 0.75.$$

The lower estimate is

$$0.2\left(\frac{1}{1+0.2}+\frac{1}{1+0.4}+\frac{1}{1+0.6}+\frac{1}{1+0.8}+\frac{1}{1+1}\right)\approx 0.65.$$

The function v is a decreasing function. So what we know is that the bug crawled more than 0.65 meters but less than 0.75 meters. Taking the average gives us a better estimate which is

$$\frac{0.65+0.75}{2}=0.7 \text{ meters.}$$

6. Problem 10
   Using the whole grid squares, we can overestimate the area as $3+3+3+3+2+1=15$ and we can underestimate the area as $1+2+2+1+0+0=6$.

7. Problem 11
   For this problem, count the squares. Each square has area 10 units so between 0 and 6 we have the area under the curve being between 140 and 150.